Surface Plasmons in Thin Metallic Films

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Abstract

For the first time it is shown that for thin metallic films thickness of which not exceed thickness of skin – layer, the problem of description of surface plasma oscillations allows analytical solution by arbitrary ratio between length of electrons free path and thickness of a film. The dependance of frequency surface plasma oscillations on wave number is carry out.

Key words: degenerate collisional plasma, surface plasma oscillations, thin metallic film.

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Introduction

Electromagnetic properties of metal films already in a current long time are a subject of steadfast attention [1] - [5]. Recently special interest involves in itself a problem about surface plasma oscillations [6] - [13]. It is connected as with theoretical interest to this problem, and with numerous practical appendices as well. Thus the majority of researches is founded on the description of properties of films with use of methods macroscopical electrodynamics. For thin films such approach is inadequate, as for the description of films in the thickness of an order and less than length of mean free path of electrons macroscopical electrodynamics is inapplicable. The electrons scattering on

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a surface demands kinetic consideration. It is serious complicates the problem.

In the present work it is shown that for thin films, a thickness of which does not exceed a thickness of a skin – layer, the problem of description of surface plasma oscillations allows the analytical solution by arbitrary ratio between length of mean free path of electrons and thickness of a film..

Let's notice, that the most part of reasonings carrying out below is fair for more general case of conducting medium (in particular, semiconductor) films.

Statement problem

Let's consider a thin metal film.

We take Cartesian coordinate system with origin of coordinates on one of the surfaces of a slab, with axes x, directed deep into the slab and perpendicularly to the surface of a film. The axes z we will direct along a direction of propagation of the surface electromagnetic wave. We will notice, that in this case a magnetic field is directed along an axis y.

At such choice of system of coordinates the electric field vector and magnetic field vector have the following structure

$$\mathbf{E} = \{ E_x(x, z, t), 0, E_z(x, z, t) \}, \quad \mathbf{H} = \{ 0, H_y(x, z, t), 0 \}.$$

The origin of coordinates we will place on the bottom plane limiting a film. Let's designate a thickness of a film through d.

Out of the film the electromagnetic field is described by the equations

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \Delta \mathbf{E} = 0$$

and

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} - \Delta \mathbf{H} = 0.$$

Here c is the velocity of light, Δ is the Laplace operator.

The solution of these equations decreasing on infinity, looks like

$$\mathbf{E} = \begin{cases} \mathbf{E}_1 e^{-i\omega t + \alpha x + ikz}, & x < 0, \\ \mathbf{E}_2 e^{-i\omega t + \alpha(d - x) + ikz}, & x > d, \end{cases}$$
(1a)

and

$$\mathbf{H} = \begin{cases} \mathbf{H}_1 e^{-i\omega t + \alpha x + ikz}, & x < 0, \\ \mathbf{H}_2 e^{-i\omega t + \alpha(d - x) + ikz}, & x > d. \end{cases}$$
 (1b)

Here ω is the frequency of wave, k is the number wave, damping parameter α is connected with these quantities by relation

$$\alpha = \sqrt{k^2 - \frac{\omega^2}{c^2}},\tag{2}$$

 \mathbf{E}_j and \mathbf{H}_j (j=1,2) are constant amplitudies.

Further components of intensity vectors electric and magnetic fields we search in the following form

$$E_x(x,z,t) = E_x(x)e^{-i\omega t + ikz}, \quad E_z(x,z,t) = E_z(x)e^{-i\omega t + ikz},$$

and

$$H_y(x,z,t) = H_y(x)e^{-i\omega t + ikz}$$

Then behaviour of electric and magnetic fields of the wave in the film is described by the following system the differential equations [3]

$$\begin{cases}
\frac{dE_z}{dx} - ikE_x + \frac{i\omega}{c}H_y = 0, \\
\frac{i\omega}{c}E_x - ikH_y = \frac{4\pi}{c}j_x, \\
\frac{dH_y}{dx} + \frac{i\omega}{c}E_z = \frac{4\pi}{c}j_z.
\end{cases}$$
(3)

Here \mathbf{j} is the current density.

The equations (3) are satisfied and out of the film under the condition $\mathbf{j} = 0$.

Impedance on the bottom surface of the layer (film) then is defined as follows

$$Z = \frac{E_z(-0)}{H_y(-0)}. (4)$$

We consider in the given work the case, when z – component of electric field has the antisymmetric configuration concerning of the film middle. Then [15] y – component of magnetic field and x – component of electric field have the symmetric configuration concerning of the film middle. Thus

$$H_y(0) = H_y(d), \qquad E_x(0) = E_x(d), \qquad E_z(0) = -E_z(d).$$
 (5)

It is required to find a spatial dispersion of the surface plasmon, i.e. to find dependence of frequency of oscillations own mode of system (3) on quantity of the wave vector $\omega = \omega(k)$.

Surface plasmon

Let's consider the case when the width of a layer d is less than depth skin – layer δ . We will notice that depth skin – layer essentially depends on frequency of radiation, monotonously decreasing in process of growth last. The quntity δ accepts the minimal value in so-called infra-red case [16]

$$\delta_0 = \frac{c}{\omega_n},$$

where ω_p is the plasma frequency.

For typical metals [16] $\delta_0 \sim 10^{-5}$ cm.

Thus for the films which thickness d is less δ_0 , our assumption holds for any frequencies.

Quantities H_y and E_z a little vary on distances smaller than depths of skin – layer. Therefore at performance of the given assumption $(d < \delta_0)$ this field will vary a little in the layer.

Let's consider the first of conditions (5) $H_y(0) = H_y(d)$. Because of this condition it is possible to accept, that the quantity H_y is constant

in the layer. Change of quantity z – projection of electric field on the thickness of the layer can define from the first equation of system (3)

$$E_z(d) - E_z(0) = -\frac{i\omega}{c}dH_y + ik\int_0^d E_x dx.$$
 (6)

From the second equation of system (3) taking into account a non-flowing condition of a current through boundary of the film and a continuity condition of electric and magnetic fields follows, that on film border holds the relation

$$E_x(0) = E_x(d) = \frac{ck}{\omega} H_y. \tag{7}$$

The integral entering into the relation (6) is proportional to value of quantity of normal to the surface of component of electric field on surfaces, and consequently to the quantity H_y . Therefore it is natural to enter proportionality coefficient

$$G = \frac{1}{E_x(0)} \int_0^d E_x(x) dx.$$

For the case $kl \ll 1$ the quantity G can be calculated from the problem about behaviour of a plasma layer in variable electric field, perpendicular to the surface layer [17].

Taking into account (7) this coefficient will be copied in the form

$$G = \frac{1}{H_y\left(\frac{ck}{\omega}\right)} \int_0^d E_x(x)dx. \tag{8}$$

Hence, expression (6) with use (8) then can be written down as

$$E_z(d) - E_z(0) = ikdH_y \left(1 - \frac{ck}{\omega}G\right).$$

Considering antisymmetric character z – projection of electric field E_z in this case we receive

$$E_z(0) = ik\frac{d}{2}H_y\left(1 - \frac{ck}{\omega}G\right). \tag{9}$$

According to (9) for an impedance (4) we have

$$Z = ik\frac{d}{2}\left(1 - \frac{ck}{\omega}G\right). \tag{10}$$

From the third equation of system (3) taking into account relations (1) we receive the following connection between y – projection of magnetic field and z – projection of electric field in the immediate vicinity from the bottom surface of the layer and out of it (when $j_z = 0$)

$$\alpha H_y(0) = -\frac{i\omega}{c} E_z(0).$$

From here we receive following expression for the surface impedance

$$Z = \frac{i\alpha c}{\omega}. (11)$$

Equating expressions (10) and (11), we receive

$$\frac{\alpha c}{\omega} = \frac{kd}{2} \left(1 - \frac{ck}{\omega} G \right). \tag{12}$$

The expression (11) can be transformed according the relation (2) to the form

$$\sqrt{k^2 - \frac{\omega^2}{c^2}} = \frac{\omega k d}{2c} \left(1 - G \frac{ck}{\omega} \right). \tag{13}$$

The equation (13) is the dispersion equation, from solution of which we find the connection $\omega = \omega(k)$.

In general case the function G, entered by the relation (8), is the function of two variables: $G = G(\omega, k)$. Therefore the dispersion equation (13) represents the difficult transcendental equation.

Let's consider further a case of low frequencies. We take such frequencies that essentially are less than frequency a volume plasma resonance of metal. In this case $|G| \ll 1$. Then the dispersion equation (13) is possible to transform to the following kind

$$(ck)^2 - \omega^2 = \frac{\omega^2 k^2 d^2}{4}.$$

The solution of this equation we will write as

$$\omega^2 = \frac{4(ck)^2}{4 + k^2 d^2},$$

hence

$$\omega(k) = \frac{ck}{\sqrt{1 + \left(k\frac{d}{2}\right)^2}}.$$

At small values of a wave vector k when $kd \ll 1$, from here we receive

 $\omega(k) = ck \left(1 - \frac{k^2 d^2}{8}\right).$

Conclusion

In the present work the dispersion relation for surface plasmon is deduced. We consider the case of an antisymmetric configuration of z – component of the electric field, directed lengthways propagation of an electromagnetic wave, and symmetric y – component of a magnetic field and x – component of electric field.

REFERENCES

- [1] Jones W.E., Kliewer K.L., Fuchs R. Nonlocal theory of the optical properties of thin metallic films // Phys. Rev. 1969. Vol. 178. No. 3. P. 1201–1203.
- [2] Kliewer K.L., Fuchs R. Optical propertues of an electron gas: Further studies of a nonlocal description// Phys. Rev. 1969. Vol. 185. No. 3. P. 805 913.
- [3] Kondratenko A.N. Penetration of waves in plasma. M: Atomizdat, 1979. 232 P. (in russian).

- [4] Antonets I.V., Kotov L.N., Nekipelov S.V., Karpushov E.N. Conducting and reflecting properties of thin metal films// Technical Physics, vol. 49, issue 11, pp. 1496-1500.
- [5] Paredes-Juárez A., Días-Monge F., Makarov N.M., Péres-Rodrígues F. Nonlocal effects in the electrodynamics of metallic slabs. JETP Lett, 90:9 (2010), 623–627.
- [6] Economou E. N. Surface Plasmons in Thin Films // Phys. Rev. 1969. V. 182. No. 2. P. 539–554
- [7] Raether H. Surface Plasmons on Smooth and Rough Surfaces and on Gratings. Springer, Berlin, 1988. P. 133. Published 1988 by Springer-Verlag in Berlin, New York.
- [8] Chang R., Chiang H.P., Leung P.T., Tse W.S. Nonlocal electrodynamic effects in the optical excitation of the surface plasmon resonance // Optics Communications. V. 225. 2003. P. 353—361.
- [9] Maaroof A.I., Gentle A., Smith G.B., Cortie M.B. Bulk and surface plasmons in highly nanoporous gold films // J. Phys. D: Appl. Phys. V. 40. 2007. P. 5675—5682.
- [10] Pitarke J.M., Silkin V.M., Chulkov E.V. and Echenique P.M. Theory of surface plasmons and surface-plasmon polaritons // Rep. Prog. Phys. 2007. Vol. 70. P. 1–87.
- [11] Anttu N., Xu H.Q. Light scattering and plasmon resonances in a metal film with sub-wavelength nano-holes // J. Phys.: Conference Series. V. 100. 2008. P. 1-4. 052037
- [12] Cade N. I., Ritman-Meer C. T., Richards D. Strong coupling of localized plasmons and molecular excitons in nanostructured silver films // arXiv:0904.2674v1 [cond-mat.mes-hall]. 2009. 4 P.

- [13] Apostol M., Vaman G. Plasmons and polaritons in a semi-infinite plasma and a plasma slab // arXiv:0904.2662v1 [physics.optics]. 2009. 20 P.
- [14] Landau L.D., Lifshits E.M. Electrodynamics of Continuous Media, Butterworth-Heinemann (Jan 1984). P. 460.
- [15] Fuchs R., Kliewer K.L., Pardee W.J. Optical properties of an ionic crystal slab // Phys. Rev. 1966. V. 150. No. 2. P. 589–596.
- [16] Lifshits E.M., Pitaevskii L.P. Physical Kinetics, Butterworth-Heinemann (Jan 1981). P. 625.
- [17] Latyshev A.V., Yushkanov A.A. Plasma in a Metal Layer Exposed to an RF Electric Field// Technical Physics, 2008, Vol. 53, No. 5, pp. 562–570.